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Lecture Notes on DC Network Theory

Harmattan Semester

by

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February 4, 2017

Recommended Text: Introductory Circuit Analysis, Third Edition
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1 DC Network Theorems

When faced with more complicated electrical networks, there are methods for making the network simpler in order to make their analytical solution easier. These methods are referred to as network theorems.

2 Kirchhoff's Voltage Law (KVL)

Kirchhoff's voltage law states that *the algebraic sum of of the voltage rises and drops around a closed loop or path is zero.* A closed loop is any continuous connection of branches which allows us to trace a path which leaves a point in one direction and returns to that same point from another direction without leaving the circuit. In Fig 1 the continuous path that starts from point a through R_1 and comes back through E without leaving the circuit is a closed loop.

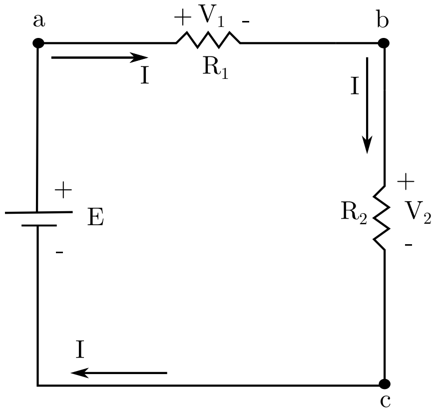

Figure 1: Kirchhoff's voltage law

Applying KVL to Fig 1 involves the addition of voltage rises and drops in one direction around the closed loop. An increase in voltage is given a positive (+ve) sign and a negative (-ve) sign is given to a decrease in voltage. This implies that when you go from the positive terminal of a battery to the negative terminal, there is a voltage drop (-ve) sign and when you go from the negative terminal of a battery to the positive terminal, there is a rise in voltage (+ve) sign. A clear illustration of these concepts can be seen in Fig 2. If we follow the current in Fig 1 from point a we get a potential drop V_1 (+ to -) across R_1 and then another potential drop V_2 across R_2. Continuing to the voltage source, we have an increase in voltage E (- to +) before coming back to point a.

In symbolic form where $\sum$ represents summation, $\circlearrowleft$ a closed loop and V the voltage

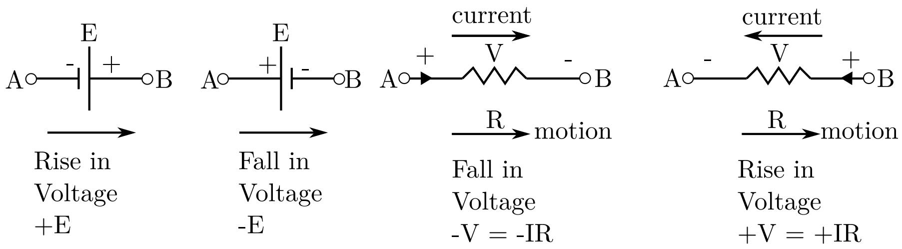

Figure 2: Determination of the sign of the voltage drop across a battery and resistor

drops and rises we have

$$\sum \circlearrowleft V = 0 \quad \text{which for Fig 1 yields} \tag{1}$$
$$-V_1 - V_2 + E = 0 \quad \text{or} \tag{2}$$
$$E = V_1 + V_2 \tag{3}$$

which shows that the voltage generated by the battery is equal to the total voltage drops within the circuit. If we take the loop in the opposite direction, we have

$$\sum \circlearrowright V = 0 \tag{4}$$
$$-E + V_2 + V_1 = 0 \quad \text{or} \tag{5}$$
$$E = V_1 + V_2 \tag{6}$$

2.1 Example 1

For the circuit in Fig 3 find the total resistance, current and unknown voltage drops. Furthermore, verify Kirchhoff's voltage law around the closed loop.

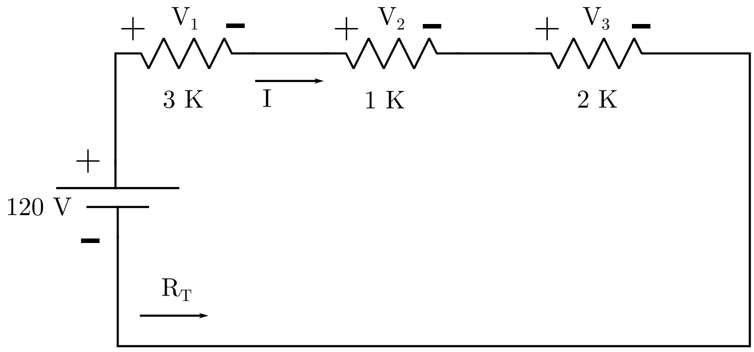

Figure 3: Example 1

Solution: The total resistance is obtained by summing up the resistances in the circuit (since they are in series) hence :

$$R_T = 3\,K + 1\,K + 2\,K = \mathbf{6\,K} \tag{7}$$

The total current is obtained by applying Ohm's law

$$I = \frac{V}{R} = \frac{120\,V}{6\,K} = \mathbf{20\,mA} \tag{8}$$

In order to verify Kirchhoff's voltage law for the circuit, we go in a clockwise loop around the circuit starting from the battery. This yields the relation

$$+120\,V - V_1 - V_2 - V_3 = 0 \quad \text{substituting for I and R we have} \tag{9}$$
$$+120\,V - 60\,V - 20\,V - 40\,V = 0 \tag{10}$$
$$+120\,V - 120\,V = 0 \quad \text{which confirms KVL} \tag{11}$$

3 Kirchhoff's Current Law

Kirchhoff's current law states that *the algebraic sum of the currents entering and leaving a node is zero.* This is symbolically given as

$$\sum I_{entering} = \sum I_{leaving} \tag{12}$$

From Fig 4 and assuming incoming currents to be positive and outgoing currents to be negative, when we consider node A:

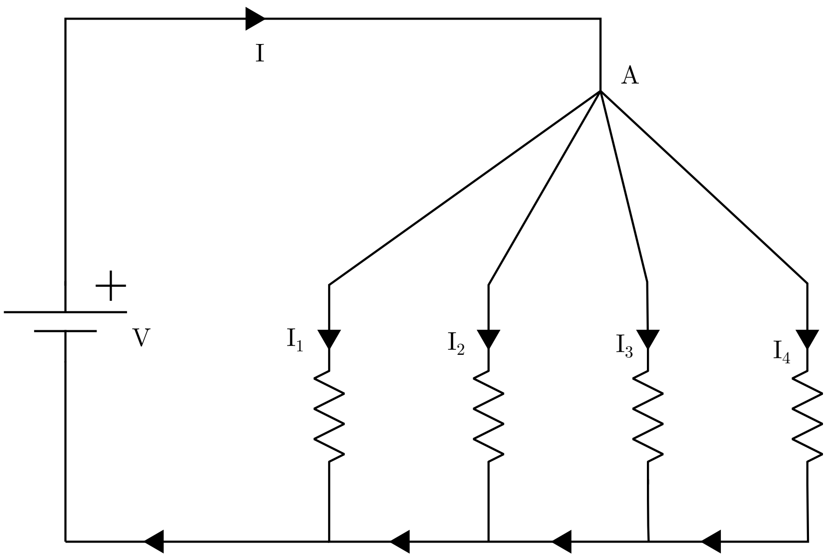

Figure 4: Kirchhoff's current law

$$+I + (-I_1) + (-I_2) + (-I_3) + (-I_4) = 0 \quad \text{so that} \tag{13}$$
$$I = I_1 + I_2 + I_3 + I_4 \tag{14}$$

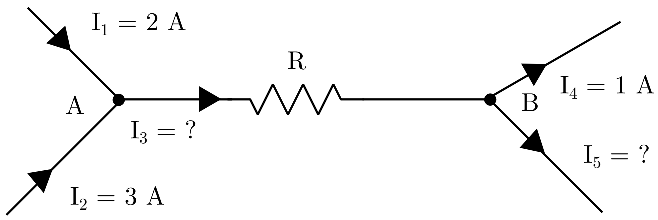

Figure 5: Example 2

3.1 Example 2

Determine the currents I_3 and I_5 in Fig 5 using Kirchhoff's current law.
Solution: Node A

$$I_1 + I_2 = I_3 \tag{15}$$
$$2 + 3 = I_3 \quad \text{and} \tag{16}$$
$$I_3 = 5\,A \tag{17}$$

Solution: Node B

$$I_3 = I_4 + I_5 \tag{18}$$
$$I_5 = I_3 + I_4 = 5 - 1 \tag{19}$$
$$I_5 = 4\,A \tag{20}$$

4 Mesh Analysis

This is one of a series of general methods employed in determining the solution for networks with two or more sources. Mesh analysis is employed to solve a circuit using the following systematic approach.

1. Assign a distinct current in a chosen direction (usually clockwise but this doesn't matter) to each independent closed loop of the network.

2. Indicate the polarities within each loop for each resistor as determined by the assumed direction of loop current for that loop.

3. Apply Kirchhoff's voltage law around each closed loop.
   a) If a resistor has two or more assumed currents flowing through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing in the same direction, minus the assumed currents through in the opposite direction.
   b) The polarity of a voltage is unaffected by the loop currents passing through it.

4. Solve the resulting simultaneous linear equations for the assumed loop currents using determinants.

4.1 Example 3

We can now apply this systematic procedure to the network in Fig 6.

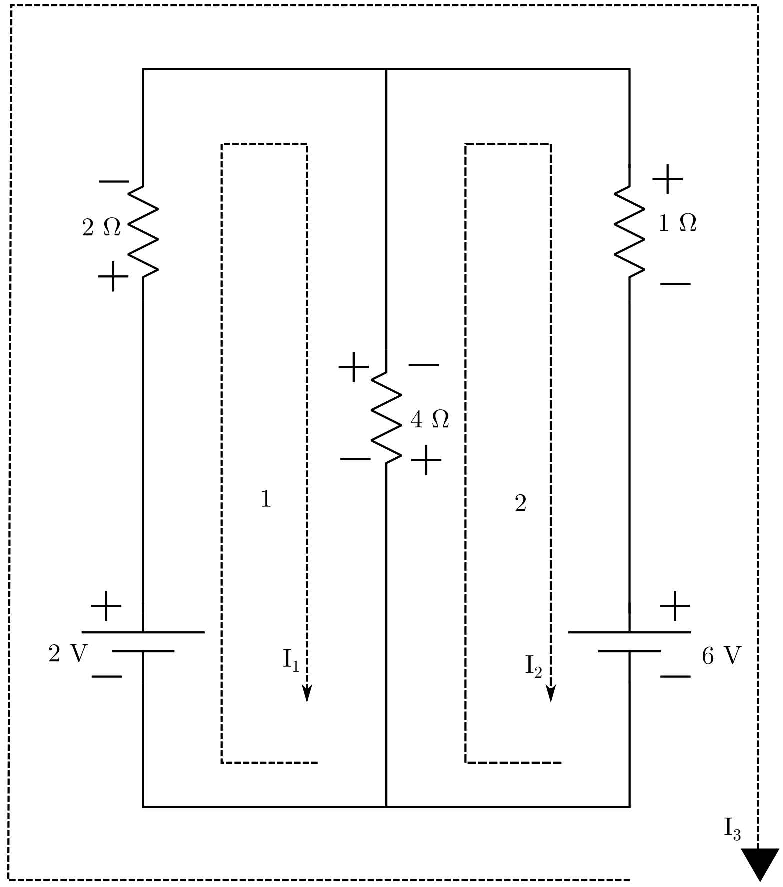

Figure 6: Mesh analysis example

Solution:

Step 1: Two loop currents (I_1, I_2) are assigned in the clockwise direction.

Step 2: Polarities are drawn to agree with assumed current directions. Note that the polarities for the 4Ω resistor are opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied for each loop:

$$\text{loop 1:} \qquad +2 - 2I_1 - 4(I_1 - I_2) = 0 \tag{21}$$

$$\text{loop 2:} \qquad -6 - 1I_2 - 4(I_2 - I_1) = 0 \tag{22}$$

Step 4: The equations are rewritten as follows:

$$\text{loop 1:} \qquad +2 - 2I_1 - 4I_1 + 4I_2 = 0 \tag{23}$$
$$\text{loop 2:} \qquad -6 - 1I_2 - 4I_2 + 4I_1 = 0 \tag{24}$$
$$\text{loop 1:} \qquad +2 - 6I_1 + 4I_2 = 0 \tag{25}$$
$$\text{loop 2:} \qquad -6 - 5I_2 + 4I_1 = 0 \tag{26}$$

Applying determinants to the above equations gives the answer:
$I_1 = -\mathbf{1\,A}$ and $I_2 = -\mathbf{2\,A}$.
The negative sign indicates that the current in the network flows in the direction opposite to that indicated by the assumed loop current. The current through the 4Ω resistor is:

$$I_1 - I_2 \qquad \text{from loop 1} \tag{27}$$
$$-1 - (-2) = -1 + 2 = \mathbf{1\,A} \tag{28}$$

5 Nodal Analysis

In mesh analysis, the general network equations are obtained by applying Kirchhoff's voltage law around each closed loop. Nodal analysis employs Kirchhoff's current law. A *node* in a network is defined as a junction of two or more branches. If we now define one node of any network as a reference, the remaining nodes of the network will all have a fixed potential relative to this reference. The nodal analysis method is systematised as follows:

1. Convert all voltage sources to current sources
2. Determine the number of nodes in the network.
3. Pick a reference node and label each remaining node with a voltage (for example V_1, V_2, V_3....)
4. Write Kirchhoff's current law at each node except the reference
5. Solve the resulting equations for nodal voltages.

5.1 Example 4

Apply nodal analysis to the circuit in Fig 7.

Solution:
Step 1: Convert all voltage sources to current sources. This results in the circuit below. V_1 is positive with respect to the reference node. Therefore current flows away

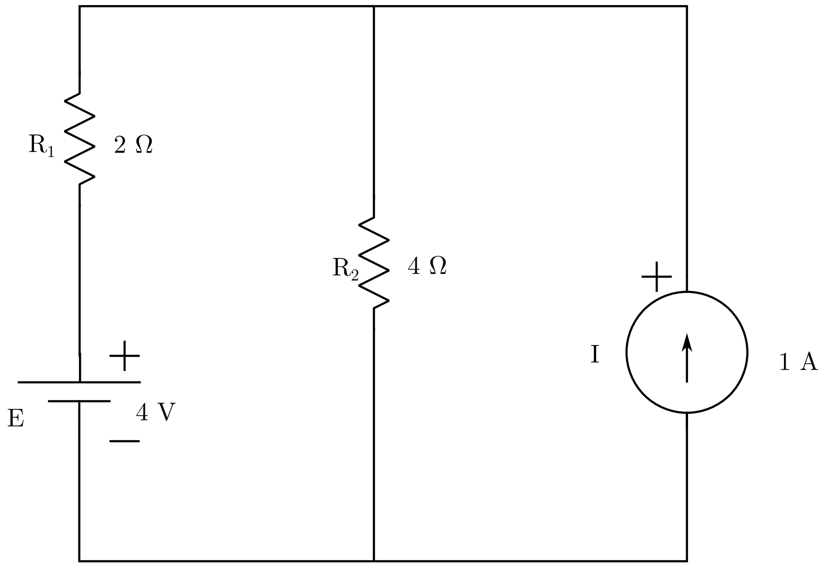

Figure 7: Example 4

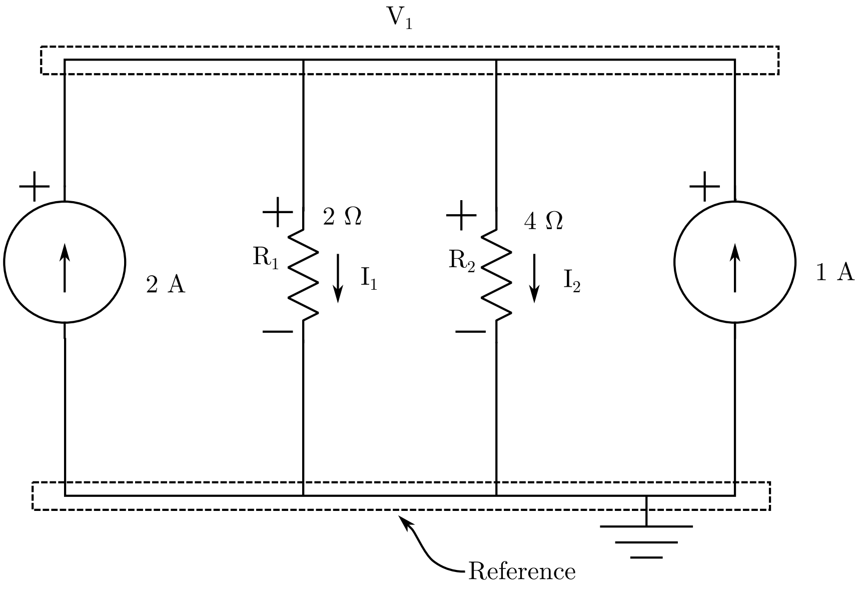

Figure 8: Example 4

from V_1 through the 2Ω and 4Ω resistors at a rate equal to:

$$I_1 = \frac{V_1}{2} \qquad \text{and} \tag{29}$$

$$I_2 = \frac{V_1}{4} \tag{30}$$

respectively. Applying Kirchhoff's current law yields:

$$2 + 1 - \left(\frac{V_1}{2} + \frac{V_1}{4}\right) = 0 \tag{31}$$

$$V_1\left(\frac{1}{2} + \frac{1}{4}\right) = 3 \qquad \text{or} \tag{32}$$

$$V_1\left(\frac{3}{4}\right) = 3 \tag{33}$$

$$V_1 = \frac{12}{3} = \mathbf{4\,V} \tag{34}$$

The potential across each current source and resistor is therefore $4\,V$ and

$$I_1 = \frac{V_1}{2} = \frac{4}{2} = \mathbf{2\,A} \tag{35}$$

$$I_2 = \frac{V_1}{4} = \frac{4}{4} = \mathbf{1\,A} \tag{36}$$

6 Superposition Theorem

Just like the previously discussed techniques, the superposition theorem can be used to analyse networks with two or more sources that are not in series. The advantage of this technique is that you do not need the use of advanced mathematical techniques like determinants to solve systems of equations.

It states that *the current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.* In other words, this theorem allows us to find a solution for a current or voltage using *only one source at a time*. Once we have the solution for each source, we can combine the results to obtain the total solution.

6.1 Example 5

To better understand this technique, consider the circuit in Fig 9. Using superposition, determine the current through the 3Ω resistor.

Solution:
First we consider the $72\,V$ and short circuit the $18\,V$ battery, resulting in Fig 10. We

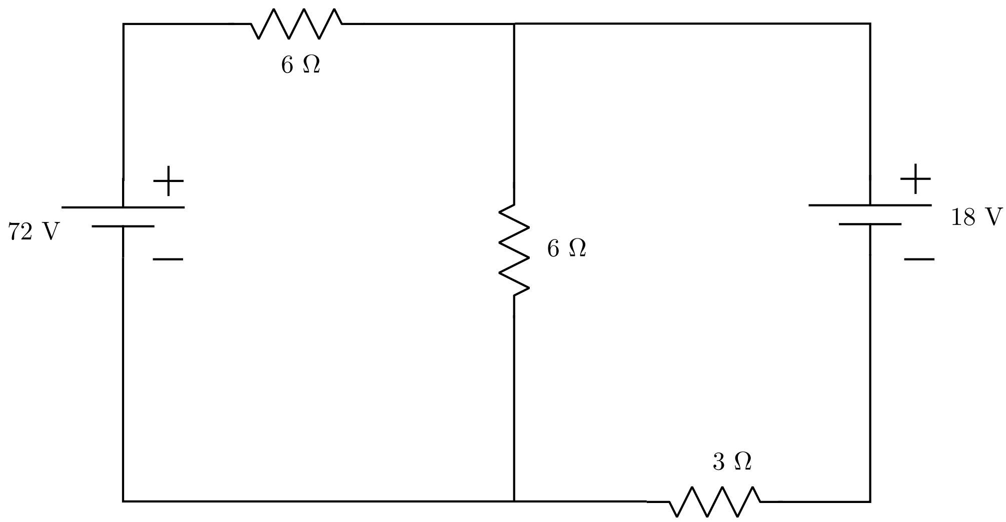

Figure 9: Example 5

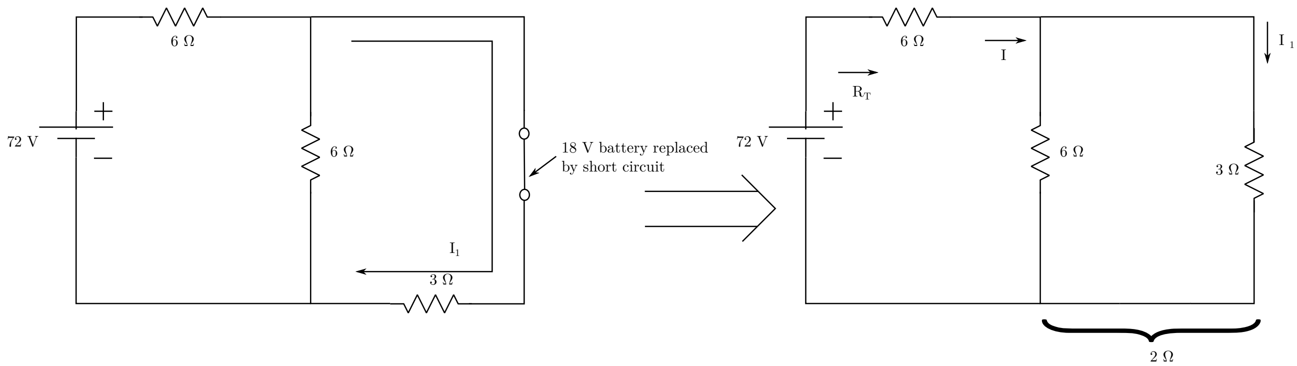

Figure 10: Example 5 contd.

can see that the $6\,\Omega$ and $3\,\Omega$ resistors are in parallel. The total resistance seen by the $72\,V$ source is therefore

$$R_T = 6\,\Omega + 6\,\Omega \parallel 3\,\Omega = 6 + \left(\frac{6 \times 3}{6+3}\right) = 8\,\Omega \tag{37}$$

The total source current I is therefore

$$I = \frac{V}{R_T} = \frac{72\,V}{8\,\Omega} = 9\,A \tag{38}$$

To obtain the current I_1 through the $3\,\Omega$ resistor we use the current divider rule

$$I_1 = 9\,A \times \left(\frac{6\,\Omega}{6\,\Omega + 3\,\Omega}\right) = 6\,A \tag{39}$$

If we now replace the $72\,V$ source with its short-circuit equivalent, the circuit in Fig 11 results We can see that the $6\,\Omega$ and $6,\Omega$ resistors are in parallel. The total resistance

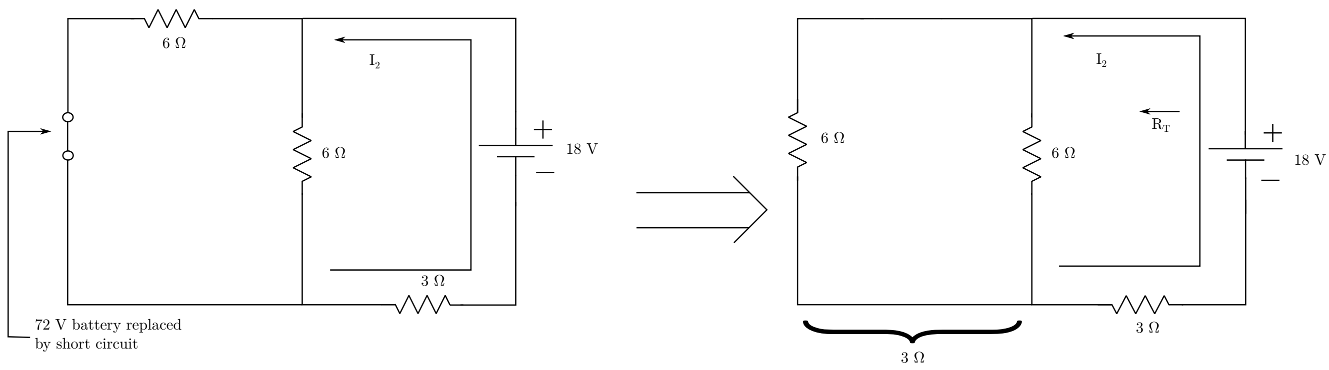

Figure 11: Example 5 contd.

seen by the $18\,V$ source is therefore

$$R_T = 3\,\Omega + 6\,\Omega \parallel 6\,\Omega = 3 + \left(\frac{6 \times 6}{6+6}\right) = 6\,\Omega \tag{40}$$

The total source current I is therefore

$$I = \frac{V}{R_T} = \frac{18\,V}{6\,\Omega} = 3\,A \tag{41}$$

This is also the current through the $3\,\Omega$ resistor since it is in series with the $18\,V$ source. Considering the effects of both sources together, the current through the $3\,\Omega$ resistor is $6\,A - 3\,A = \mathbf{3\,A}$

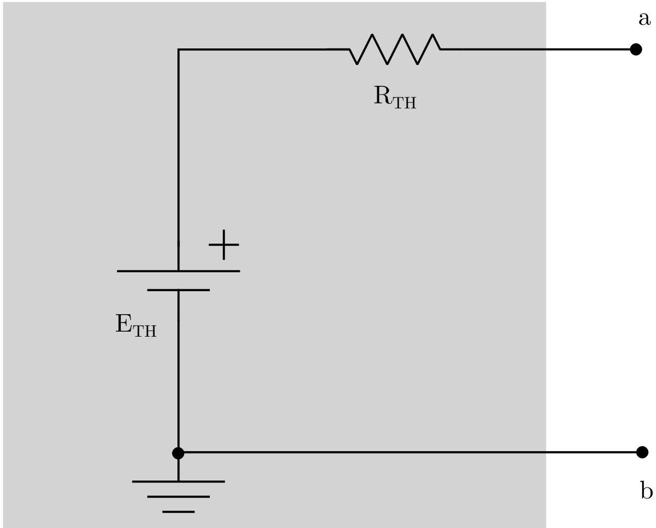

Figure 12: Thévenin equivalent circuit

7 Thévenin's Theorem

Thévenin's theorem states that *Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor* as shown in Fig 12. It is generally used in the following situations.

- Analyze networks with sources that are not in series or parallel.
- Reduce the number of components required to establish the same characteristics at the output terminals.
- Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.

To determine the Thévenin's equivalent circuit for any network, the following systematic procedures have to be adhered to

1. Remove that portion of the network where the Thévenin equivalent circuit is found. In Fig 13 this requires that the load resistor R_L be temporarily removed from the network.
2. Mark the terminals of the remaining two-terminal network.
3. Calculate R_{TH} by first setting all sources to zero (voltage sources are replaced by short circuits and current sources by open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.).

4. Calculate E_{TH} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals. (This step is invariably the one that causes most confusion and errors. In all cases, keep in mind that it is the open-circuit potential between the two terminals marked in step 2.).

5. Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit. This step is indicated by the placement of the resistor R_L between the terminals of the Thévenin equivalent circuit as shown in Fig 17.

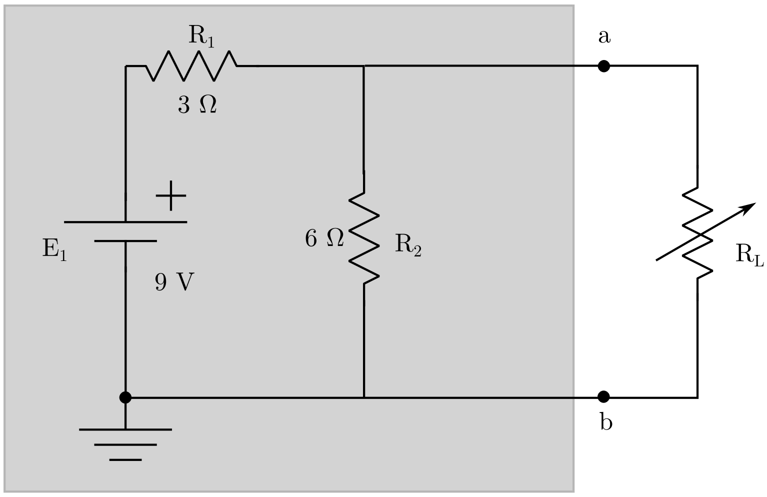

Figure 13: Thévenin example

7.1 Example

Find the Thévenin equivalent circuit for the network in the shaded area of the network in Fig 13. Then find the current through R_L for values of $2\,\Omega$, $10\,\Omega$, and $100\,\Omega$.

Solution:
Steps 1 and 2: These produce the network in Fig 14. Note that the load resistor R_L has been removed and the two "holding" terminals have been defined as *a* and *b*.

Step 3: Replacing the voltage source E_1 with a short-circuit equivalent yields the network in Fig 15 where

$$R_{Th} = R_1 \parallel R_2 = \frac{3\,\Omega \times 6\,\Omega}{3\,\Omega + 6\,\Omega} = \mathbf{2\,\Omega} \tag{42}$$

Step 4: Replace the voltage source (Fig 16). For this case, the open-circuit voltage E_{TH} is the same as the voltage drop across the $6\,\Omega$ resistor. Applying the voltage divider rule gives

$$E_{TH} = \frac{R_2 E_1}{R_2 + R_1} = \frac{6\,\Omega \times 9\,V}{6\,\Omega + 3\,\Omega} = \frac{54\,V}{9} = \mathbf{6\,V} \tag{43}$$

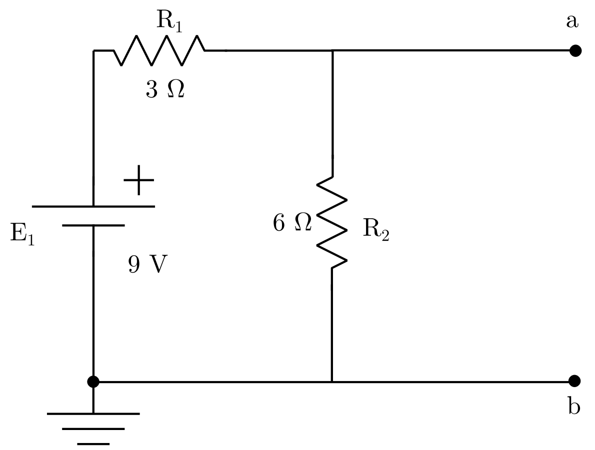

Figure 14: Example contd.

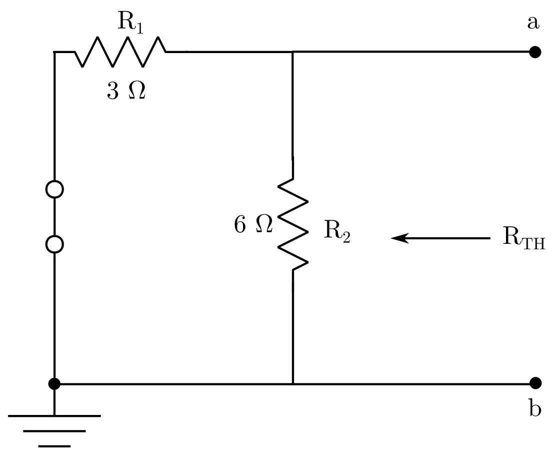

Figure 15: Example contd.

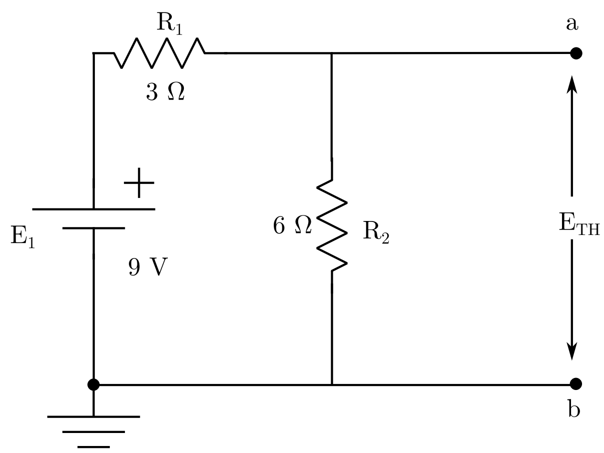

Figure 16: Example contd.

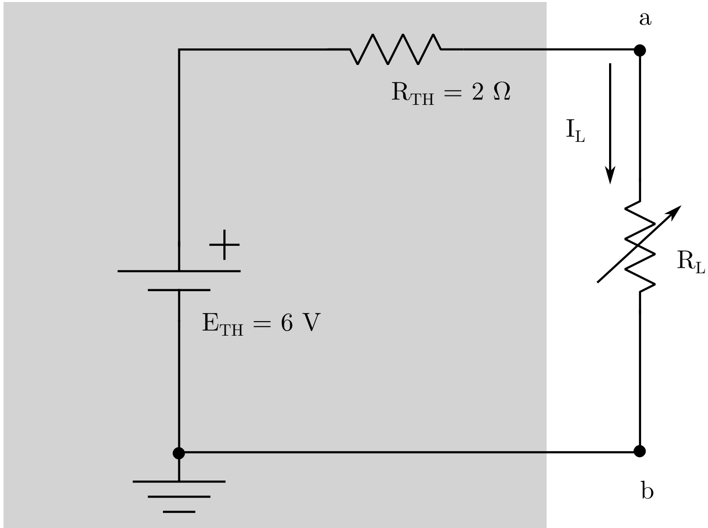

Figure 17: Example contd.

Step 5: (Fig 17):

$$I_L = \frac{E_{TH}}{R_{TH} + R_L} \tag{44}$$

$$R_L = 2\,\Omega \qquad \frac{6\,V}{2\,\Omega + 2\,\Omega} = \mathbf{1.5\ A} \tag{45}$$

$$R_L = 10\,\Omega \qquad \frac{6\,V}{2\,\Omega + 10\,\Omega} = \mathbf{0.5\ A} \tag{46}$$

$$R_L = 100\,\Omega \qquad \frac{6\,V}{2\,\Omega + 100\,\Omega} = \mathbf{0.06\ A} \tag{47}$$

8 Norton's Theorem

Norton's theorem states that *Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor.*

The series of steps involved in obtaining a Norton equivalent network are as follows:

1. Remove that portion of the network across which the Norton equivalent circuit is found.

2. Mark the terminals of the remaining two-terminal network.

3. Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{TH}$, the procedure

and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.

4. Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

5. Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

8.1 Example

Find the Norton equivalent circuit for the network in the shaded area in Fig 18.

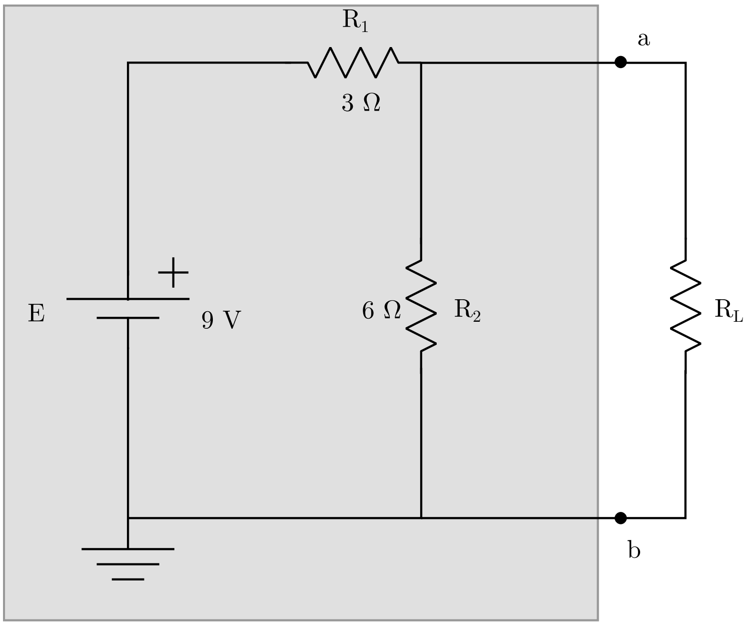

Figure 18: Example

Solution:
Steps 1 and 2: We remove the load resistance R_L and mark the terminals a and b.
Step 3: We determine the Norton resistance by short-circuiting the voltage source and determining the resistance obtained by "looking in" through the marked terminals (see Fig 19).

$$R_N = R_1 \parallel R_2 = 3\,\Omega \parallel 6\,\Omega = \frac{3\,\Omega \times 6\,\Omega}{3\,\Omega + 6\,\Omega} = \frac{18\,\Omega}{9} = \mathbf{2\,\Omega} \tag{48}$$

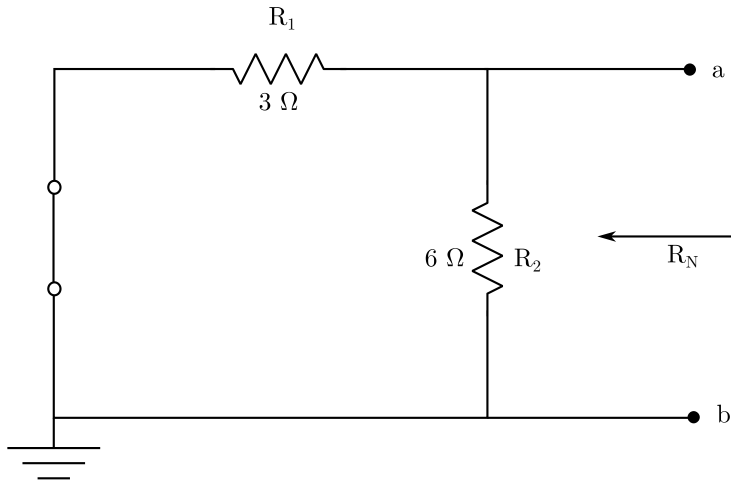

Figure 19: Example contd.

Step 4: The short-circuit connection between terminals a and b is in parallel with R_2 and eliminates its effect (see Fig 20). I_N is therefore the same as through R_1 , and the full battery voltage appears across R_1 since

$$V_2 = I_2 R_2 = 0 \times 6\,\Omega = 0\,V \quad \therefore \quad I_N = \frac{E}{R_1} = \frac{9\,V}{3\,\Omega} = \mathbf{3\,A} \tag{49}$$

Step 5: The Norton equivalent circuit is now substituted for the network external to

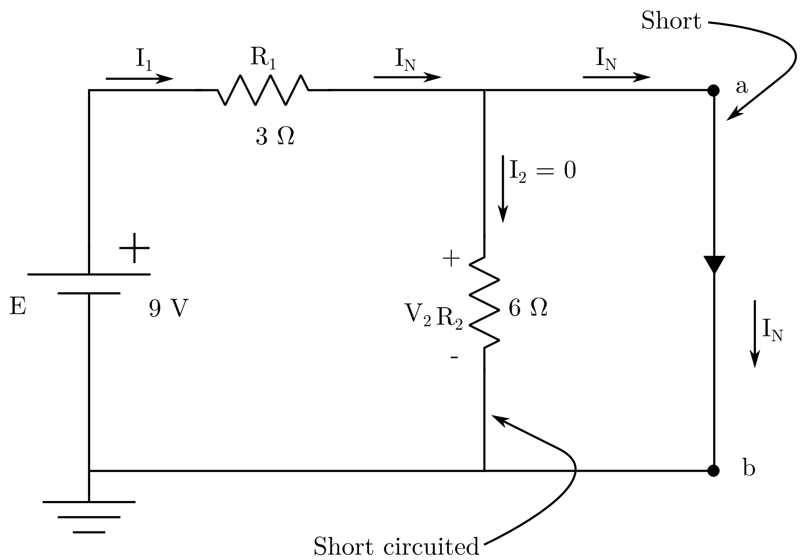

Figure 20: Example contd.

the resistor R_L in Fig 18. The result is shown in Fig 21.

It should be noted that Thévenin circuits can be converted to Norton circuits and vice-versa. Fig 22 below illustrates how this can be achieved.

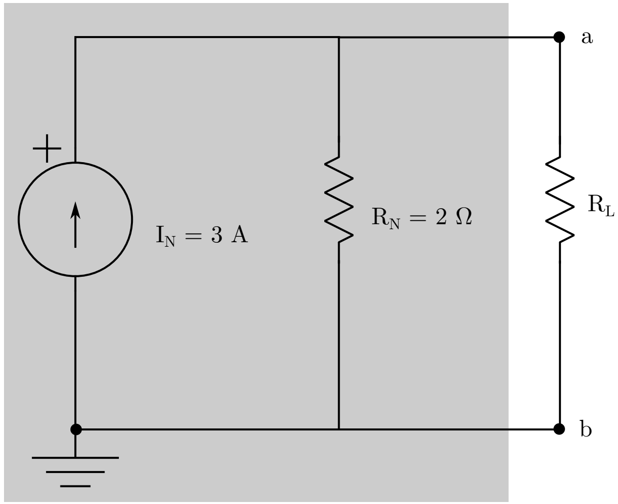

Figure 21: Norton equivalent circuit for example

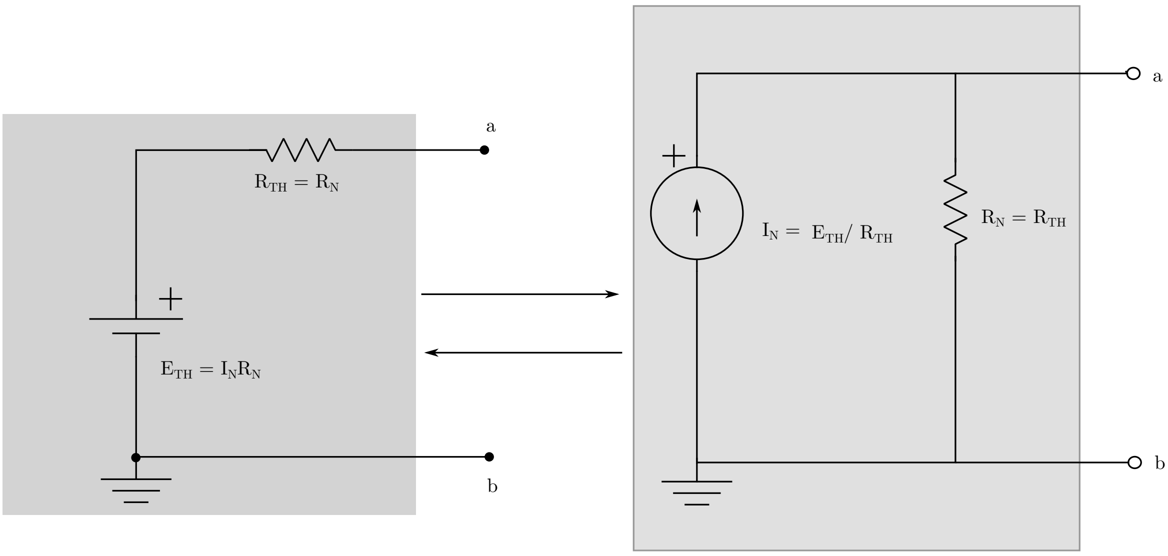

Figure 22: Conversion between Thévenin and Norton circuits